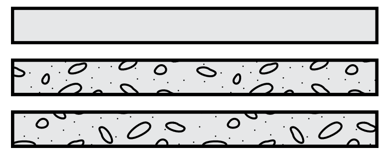
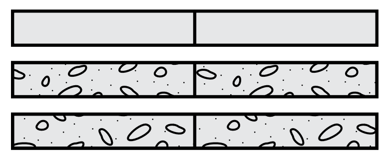
Fractions Sample Lesson

- **Equivalent Fractions**

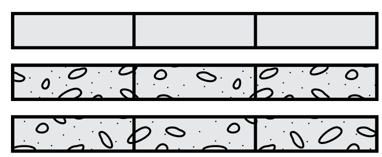
> The concept of equivalent fractions has been modeled and explained many times in previous sections.
> In the lesson, we want to show the process for finding fractions that are equivalent to a particular fraction.



A. As you can see in the picture,
there are 3 candy bars, and 2 of them have peanuts. So, $\frac{2}{3}$ of bars have peanuts.



If we cut each bar into 2 equal pieces, then
we have 6 total pieces, and 4 of them have peanuts. So, $\frac{4}{6}$ of bars have peanuts.



If we cut each bar into 3 equal pieces,
then we get 9 total pieces, and 6 of them have peanuts. So, $\frac{6}{9}$ of bars have peanuts.

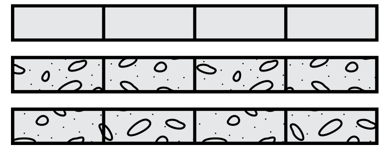
It's obvious that $\frac{2}{3}$ of the bars, $\frac{4}{6}$ of the bars, and $\frac{6}{9}$ of the bars are all the same amount of candy.

Fractions like these, which represent the same amount of something, are called equivalent fractions. So $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{6}{9}$ are equivalent fractions—they are different symbols for the same number. We can write:

$$\frac{2}{3} \text{ bars } = \frac{4}{6} \text{ bars } = \frac{6}{9} \text{ bars}$$

B. Notice that when we cut each bar into some number of equal parts, we are making more total parts. (We are multiplying.)

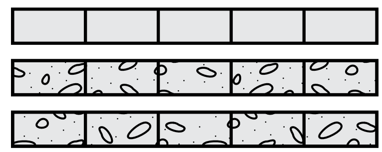
If we cut each of the 3 bars into 2 parts, we get 2×3 total parts.
If we cut each of the 3 bars into 3 parts, we get 3×3 total parts.
If we cut each of the 3 bars into 4 parts, we get 4×3 total parts.



C. Now suppose we cut each candy bar into 4 equal parts. Then there are 4 × 3 (or 12) total parts, and 4 × 2 (or 8) of them have peanuts. So, $\frac{8}{12}$ of the bars have peanuts.

This means that $\frac{8}{12}$ of the bars is the same amount of candy as $\frac{2}{3}$ of the bars, $\frac{4}{6}$ of the bars, and $\frac{6}{9}$ of the bars. These fractions are all <u>equivalent</u> to each other.

D. Could we cut each bar into 5 pieces, or 10 pieces, or any number of pieces we want? Of course we could!



In this picture, each bar has been cut into 5 equal pieces. This makes 5 × 3 (or 15) total pieces, and 5 × 2 (or 10) of them have peanuts. So, $\frac{10}{15}$ of bars have peanuts.

This gives us another fraction which represents the same amount of candy as $\frac{2}{3}$ or $\frac{4}{6}$ or $\frac{6}{9}$ or $\frac{8}{12}$ of the bars.

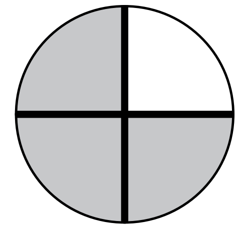
E. There is a clear pattern in all of this which we can see when we write the numerators and denominators of the fractions as <u>products</u>. This is a list of the fractions that are equivalent to $\frac{2}{3}$:

$\frac{2}{3}$,	$\frac{2\times2}{2\times3}$,	$\frac{3\times2}{3\times3}$,	$\frac{4\times2}{4\times3}$,	$\frac{5\times2}{5\times3}$, …	$\frac{10\times2}{10\times3}$	, … on and on "forever"
	↑ bars cut into 2 equal pieces	↑ bars cut into 3 equal pieces	↑ bars cut into 4 equal pieces	↑ bars cut into 5 equal pieces	↑ bars cut into 10 equal pieces	

In ordinary form, the fractions look like this:

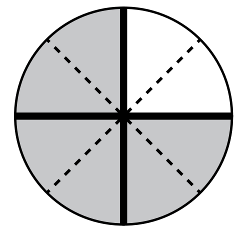
$$\frac{2}{3}\ ,\quad \frac{4}{6}\ ,\quad \frac{6}{9}\ ,\quad \frac{8}{12}\ ,\quad \frac{10}{15}\ ,\ \cdots\quad \frac{20}{30}\ ,\ \cdots$$

F. Suppose we were asked what fraction of this circle is shaded. We would probably all say, $\frac{3}{4}$—because that's what we "see" when we look at the picture.



But suppose we think about cutting each of the fourths of the circle into 2 equal shares.

Then, our mental picture looks like the one at right, and we see the shaded part as $\frac{3}{4}$ or $\frac{6}{8}$.



So $\frac{3}{4}$ of the circle $=$ $\frac{6}{8}$ of the circle.

This means that $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions.

Now suppose that the original 4 parts of the circle were cut into 3 equal pieces.

i. Draw a picture to show what the circle looks like now.

ii. How many total equal pieces are there? How many of them are shaded? Is $\frac{9}{12}$ equivalent to $\frac{3}{4}$?

iii. Write the next equivalent fraction in each list.

$\frac{3}{4}$, $\frac{2\times3}{2\times4}$, $\frac{3\times3}{3\times4}$, ________

$\frac{3}{4}$, $\frac{6}{8}$, $\frac{9}{12}$, ________

G. a. Write four different fractions which are all equivalent to $\frac{1}{2}$.

b. Write four different fractions which are all equivalent to $\frac{1}{3}$.

c. Write four different fractions which are all equivalent to $\frac{2}{5}$.